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The Use of Multiplying Factors in Radiation Transfer Iterations

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Shock structures present interesting dispersion characteristics when the radiative exchanges involve a large fraction of the total energy. If a large part of the emitted radiation escapes upstream, highly cooled gases appear behind the shock and the final thermodynamic state of the medium is not overly affected by the passage of even very strong shocks. A numerical technique was developed by Chapin for the analysis of such radiative flows and is described here.

INTRODUCTION

Recent interest in shortened mission times for deep space probes has motivated new studies of reentry phenomena at extremely high speeds (15 km/sec and more). Typical examples have been discussed in a recent review [1]. Such reentry flows present two outstanding features:

1. Because of the strong temperature dependence of radiation emission processes, the radiation-convection ratio tends to increase rapidly with velocity: radiation becomes the dominant transfer mechanism in the fluid mechanics of the flow across the shock layer.

2. The high shock layer temperatures tend to produce a large number of high frequency photons which have enough energy to ionize the cold ground state atmospheric particles ahead of the shock. The resulting ionization zone is called an *electron precursor* or *precursor*.

This note discusses these two effects as they come out of two recent studies carried out by Chapin [2] and Nelson [3] for normal one-dimensional shocks. In particular, a technique developed by the former author to avoid numerical diver-

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gence of shock wave structures of large radiation-convection ratios[1] will be illustrated.

RADIATING SHOCK STRUCTURES

Reviews of radiating shock structure analyses have been written [4], including in particular the classic papers of Heaslet–Baldwin [5] and Ferrari–Clarke [6]. Radiation plays the part of the dispersing mechanism of such shocks (Fig. 1).

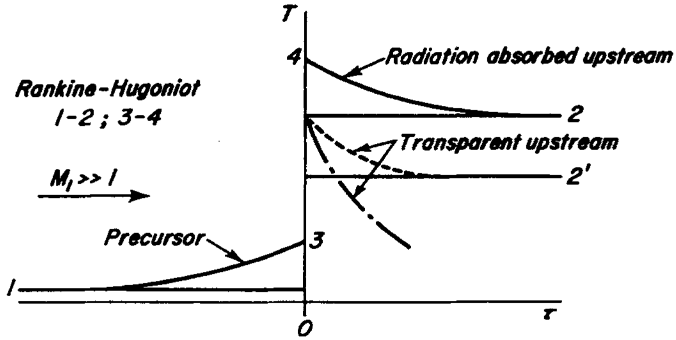

FIG. 1. Radiation exchanges across shocks.

Heaslet–Baldwin's calculations showed the role of the shock Mach number and the radiation-convection ratio for a perfect gray gas. Ferrari–Clarke extended the analysis to include both collisional and radiative rate processes, in a simplified atomic gas model with only *one* bound electronic state (this model includes, from the list on Fig. 2, the second collisional reaction and the first radiation transition only). This analysis included one additional mass conservation equation, where the electron species α was controlled by collision and radiation processes. The paper brought out the characteristic zones of collision and radiation relaxation and their relative role for varying densities.

Later, Chapin [2], followed by Nelson [3], improved further the model by allowing an excited state A* to exist alongside the ground state A. All the processes listed on Fig. 2 were included, except for the line[2] transition $h\nu_2$. Also the expected electron temperature T_e lag behind the ion–atom temperature T_a was

[1] The radiation-convection ratio in normal shock waves is also known as the inverse of the Boltzmann Number *Bo*: $Bo^{-1} \equiv q_0^R / \frac{1}{2}\rho_0 u_0^3$, where q_0^R, ρ_0, and u_0 are typical radiation flux, density, and velocity of the flow.

[2] The advantages and inconveniences of omitting line radiation from such studies are discussed in Ref. 1.

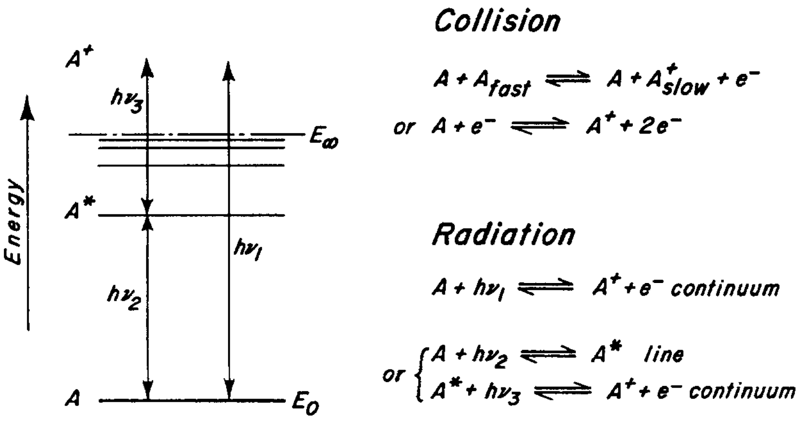

FIG. 2. Ionization processes.

accounted for, through the inclusion of a separate energy equation for the electron gas.

The system of coupled conservation equations to be solved now numbers five (Ref. 7, Eqs. 1-5): mass, momentum, total energy, electron energy, and electron species. The five unknown parameters are the density ρ, the velocity u, the ion temperature T_a, the electron temperature T_e and the degree of ionization α (i.e., the ratio of the number of electrons to the total number of particles). In non-dimensional form, these variables become:

$$\bar{\rho} \equiv \frac{\rho}{\rho_0} \qquad \bar{u} \equiv \frac{u}{u_0} \qquad \bar{T}_a \equiv \frac{T_a}{T_0} \qquad \bar{T}_e \equiv \frac{T_e}{T_0} \qquad \bar{\alpha} \equiv \frac{\alpha}{\alpha_0},$$

where the reference value (subscript 0) are usually determined from the classical Rankine–Hugoniot solution across the shock discontinuity. The solution of these equations is obtained in terms of the distance x from the shock discontinuity. Figures 3 to 5 illustrate some typical results.

Figure 3 shows results obtained by retaining the Ferrari–Clarke model ($T_e = T_a$; no radiation escape since all photons have at least enough energy ($h\nu_1$) to ionize the cold particles upstream). One recognizes the existence of a precursor ahead of the shock ($\alpha > 0$), followed by a region where the rate of increase of ionization to its equilibrium value is governed by the effects of collision and radiation rates simultaneously. Because true equilibrium is eventually reached, a Rankine–Hugoniot relationship exists between infinity upstream and infinity downstream.

Figure 4 shows the changes which occur when some of the electrons are allowed to recombine to an excited state A*: they release photons of smaller frequency ($h\nu_3$), which cannot ionize the cold gas since it consists almost exclusively of

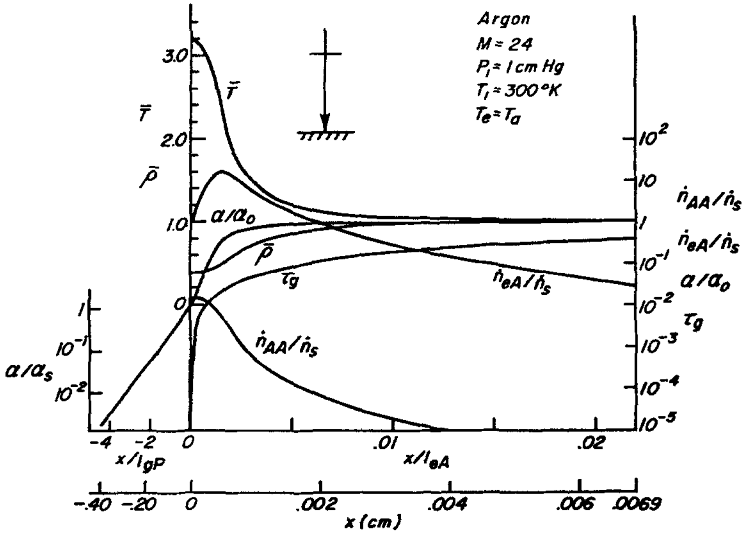

FIG. 3. Shock wave with ground-state (trapped) continuum radiation, $T_e = T_a$, $\rho = 1$ cm Hg.

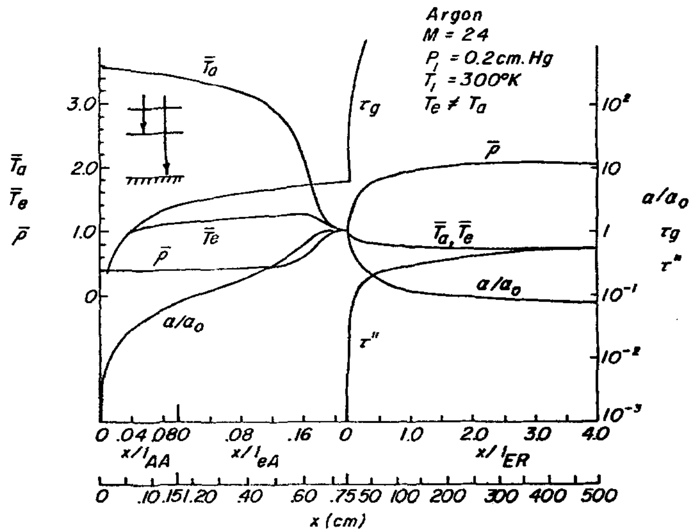

FIG. 4. Shock wave with ground- and excited-state continuum radiation, $T_e \neq T_a$, $\rho = 1$ cm Hg.

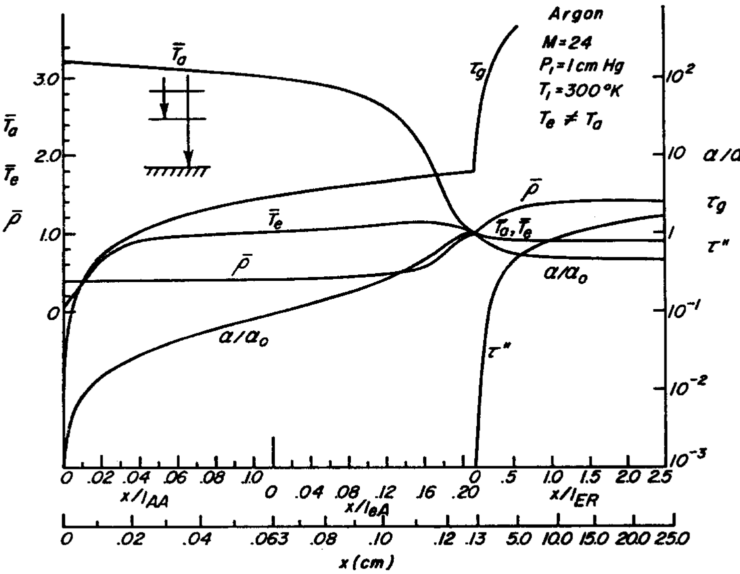

FIG. 5. Shock wave with ground- and excited-state continuum radiation, $T_e \neq T_a$, $p = 0.2$ cm Hg.

particles in their ground state A, requiring photons of larger frequency ($h\nu_1$) for ionization (Fig. 2). Thus such small frequency photons are not absorbed and the value of α reached at the end of the collision ionization period (α_0) diminishes further downstream by about one half, due to these radiation losses ($h\nu_3$) to infinity upstream. One notes also that the effect of slow electrons behind the shock ($T_e \ll T_a$) is to absorb rather than emit ground-state continuum radiation behind the shock, which effectively inhibits the precursor.

Figure 5 presents a nearly identical case[3] to the preceding one, except that the ambient pressure has been reduced by a factor of 5. As could be expected, the x scale of the processes has marquedly increased since both collisions and absorption are density dependent. Also, the radiation-convection ratio is much larger: 91.69 instead of 23.03 (see Ref. 7, Table I). Consequently, radiation processes are likely to interact more strongly with the various energy states of the gas. In particular, this results in much more radiation cooling by electron recombination, as is shown by the low degree of ionization α at the end of the process: less than a tenth of the maximum value α_0.

This last point is quite important in the sense that it gives support to Whitney–Skalafuris' assumption [8] that shocks propagating through stellar atmospheres

[3] In order to keep the same Mach number ($M = 24$) and upstream temperature ($T = 300°K$) at this reduced pressure, we introduced a change of shock velocity (from 6.58×10^4 to 7.23×10^4 cm/sec).

(radiation-convection ratios of about 10^3 or 10^4) are fairly transparent to their own longer wave radiation (e.g., Balmer continuum) and leave a relatively unheated thermodynamic state in their wake.

MULTIPLYING FACTORS

The analysis of large radiation-convection ratio cases presents an interesting problem of numerical nature, which was solved by Chapin (see pp. 71–80 of Ref. 2)[4]

In general, the iterative solution of a shock structure, for a given ambient state and shock velocity, begins with the assumption of a radiationless solution. The precursor which corresponds to the flux emitted by this particular shock structure can be then calculated in closed form with good approximation (the cool absorbing region can be modelled rather simply). The Rankine–Hugoniot conditions are then applied at the shock discontinuity and a marching downward process is then initiated from this point immediately behind the shock. At each step of this march, values from the preceding solution are used whenever downstream information is needed (such as, for instance, the flux received at this point from the layers downstream of it). This procedure eventually yields a first approximation to all the properties behind the shock. A new precursor is then calculated on this basis, and the process performed again until convergence of all profiles is obtained.

In such a problem, the main difficulty comes from the fact that the photons absorbed at any point of the flow are contributed by the entire flow field. Therefore, if we start (1st approximation) with a radiationless solution where the ionization α stays large behind the shock (no radiation cooling losses), the radiative flux calculated from this first approximation will likewise be very large. It will cause the gas to cool (on paper) much faster than expected in reality. The resulting overcooled solution (2nd approximation) will yield a very small radiation field on the next run and the computations will now yield a near equilibrium gas solution behind the shock (3rd approximation), since there is practically no radiation field. And so on... No solution is likely to emerge from this strongly oscillating sequence.

As a means to avoid such oscillations, controllable multiplying factors were adopted for the sensitive excited state continuum flux terms $\bar{Q}_{OM}$ and $\bar{Q}'_{OM}$ (i.e., the reduced flux $\bar{Q}_{OM}$ itself and the flux dependent function $\bar{Q}'_{OM}$ which enters the

[4] During the presentation of this note to the Novosibirsk Congress, it was pointed out to the author that a similar method has been simultaneously and independently developed for the solution of detached shock layer flows in air (Ref. 9). As the instability inherent to the present application (downstream flow of infinite extent) appears to be more severe than in Ref. 9, both contributions should be considered complementary rather than redundant. RG.

expression for the rate of ionization). These arbitrary multiplying factors could be increased progressively by small fractions from zero to unity.

In this fashion, the terms $\bar{Q}_{OM}$ and $\bar{Q}'_{OM}$, which control the radiative cooling and ionization rate respectively, could be *turned on* carefully. The rate at which these multiplying factors could be increased toward unity in successive solutions is quite sensitive to the radiative-convection ratio, and therefore to the pressure in front of the shock. For low radiation-convection ratios and for high initial pressure, collisional processes dominate the radiative processes and the factors can be rapidly increased. (For instance; 0., 0.1, 0.3, 0.6, 1.0). However, for high radiation-convection ratios and for low ambient pressures, radiative processes become more important relative to collisional processes and the factors must be increased very slowly. (For instance, by steps of 0.05 or less.)

As an illustration, the successive steps in the solution of the 0.2-cm Hg pressure case (Fig. 5) are shown on Fig. 6. The logarithm of the degree of ionization in the radiation cooling region is shown. The degree of ionization is directly affected by the excited state radiation in the radiative cooling region.

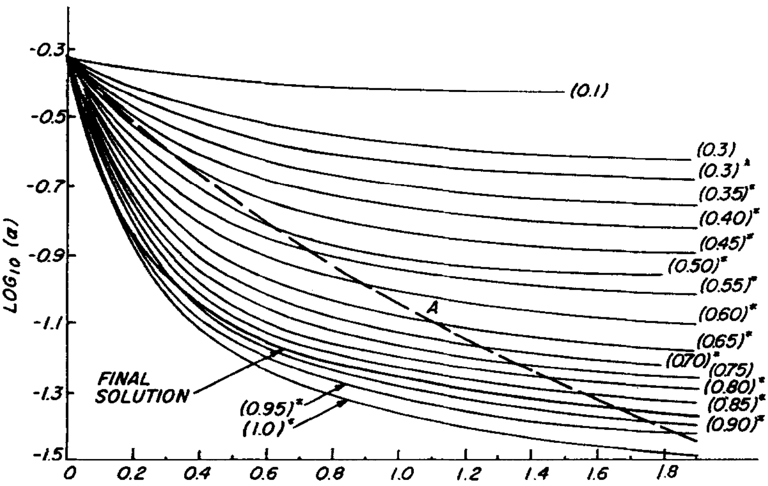

FIG. 6. Multiplying factors as applied to the iterations leading to Fig. 5.

The curve labeled 0.1 is the solution obtained with the $\bar{Q}_{OM}$ and $\bar{Q}'_{OM}$ multiplying factor set to 0.1 using the radiationless solution as the previous solution. Curve 0.3 was obtained when the factor was increased to 0.3. When the factor was increased to 0.5 the curve labeled A was obtained. This curve indicates that in this case the factor was increased much too fast: therefore, going back to the 0.3 solution, the factors $\bar{Q}_{OM}$ and $\bar{Q}'_{OM}$ were increased successively from 0.3 to 0.5 in steps of 0.05

and each solution was averaged with the previous one. The factors were then held at 0.5 and convergence was reached resulting in the curve labeled (0.50).* The factors were then increased by steps of 0.05 until the value of 1.0 was reached, each time averaging with the previous solution. With the factors equal to one, convergence was rapidly reached resulting in the curve labeled *final solution*.

CONCLUSION

High radiation-convection shocks (low Boltzmann number) present high energy circulation features which result into extensive post shock cooling when most of the emitted radiation is of frequency lower than the ionization frequency of the gas upstream. A *multiplying factor* technique has been developed, which successfully handles the numerical instabilities inherent to the large property transients across the shock.

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